



## Finding Default Barrier and Optimal Cutoff Rate in KMV Structural Model based on the best Ranking of Companies

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### ABSTRACT

According to the adverse consequences that are brought by financial distress for companies, economy and financial –monetary institutions, the use of methods that can predict the occurrence of financial failure and prevent the loss of wealth is of great importance. The major models of credit risk assessment are based on retrospective information and using the methods which use the updated market data for prediction of the probability of default can lead to the increase of the reliability of results. The purpose of this study is to obtain optimal default barrier in KMV model by using an approach based on genetic algorithm and compare the performance of the proposed model to KMV model. Research data included all data of listed companies in the Tehran stock exchange that were bankrupted from 2009 to 2014 according to the article 141 of the commercial code. In total, 25 companies were considered as distressed companies and 50 non-bankrupted companies were also selected as the control group and then results of the two models were compared. The study results showed that the performance of the presented model in prediction of bankruptcy and separating distressed from non-distressed companies is better than KMV model. At the end, the optimal cut off rate was calculated to determine whether a specific company will be bankrupt or healthy according to its probability of default. The results showed that the calculated optimal value led to 80% correct prediction in 2015.

### Keywords:

Credit Risk, KMV model, Probability of Default.

## 1. Introduction

The decisions related to granting credit was often done individually according to judgment methods in the past. These methods were time consuming and costly and they did not have scientific credibility; hence, the role of technology in the credit management process of banking and financial institutions is increasing. Due to the fact that the customers' credit measurement process is complex and also the validation plays a key role in the institutions performance success, to design validation systems is needed.

Studies about reasons behind the commercial failure of companies go back to 1930s. Predicting bankruptcy was known as one of the important research subjects in finance literature. Many academic studies attempted to present the best models of bankruptcy prediction according to the available data and statistical techniques. A large number of models have been developed to identify the companies' probability of default based on financial and non-financial data. However, using a model that can identify financial distresses before their occurrence and propose appropriate solutions has special importance. Three general categories of credit risk are: structural models, reduced form models and credit rating models. The rating model was proposed by Altman(Altman, 2000). He identified significant variables in default by studying the financial statements of companies and gave a score to each company by providing a linear combination of financial variables. Then, these credit scores were converted into probability of default.

In structural models, the company's assets value depends on the value of cash flows which will be produced in future. Debts and shareholders' equity indicate the total claims on the company's assets. Debts have higher priority and shareholders receive the remaining value. The main assumption of structural models is that a default occurs when the value of the company's assets is not enough for reimbursement of the company's debts. In the Merton's initial model (Merton, 1974) the company's debts only consist of one zero coupon bond with nominal value  $L$  and due time  $T$ . No payment is done before time  $T$ . Accordingly, the default occurs when the value of assets would be less than the debts value (Löeffler and Posch, 2011).

One of the modifying models of Black-Scholes-

Merton which practically has improved this model is known as KMV-Merton approach. KMV-Merton approach uses a new default point for calculating probability of default which is equal to the current debt plus the half of long term debts instead of the total liabilities(Bohn and Crosbie, 2003). In order to solve the problem of optimal default barrier in KMV-Merton approach, genetic algorithm is used by (Lee, 2011) to redefine optimal default barrier in this model. He did not raise any specific constraint on  $\alpha$  and  $\beta$ , so the default barrier estimated by him was more than the total liabilities of company which was contrary to the basic assumptions of Merton model. The idea of KMV model for projection of a new default barrier was completely contrary to this issue because it attempted to define a new default barrier which was less than total liabilities of companies.

In this study, to guarantee that default barrier for all companies will be located in the feasible area, some constraints were added to the model and also a new approach was provided to find optimal default barrier and resolved mentioned inaccuracies. This approach counted healthy and unhealthy companies to take false-negatives as well as false-positives into account. At the end, optimal cut off rate as a reference amount for decision-making calculated. The probability of default of companies can be compared with this and then judgment about the future of companies can be discussed. We believe there is an optimal cut off rate that can be varied from a country to another one or even from an industry to another one and it is better to be calculated separately for each market.

## 2. Literature Review

Credit risk is the uncertainty associated with the ability of companies to pay their liabilities and obligations(Committee, 1999). In other words, according to this risk, reimbursements are done with delay or are not collected at all. It causes some problems in the cash of banks. Despite the innovations in financial services sector, this type of risk is still considered as the major cause of failure in financial and credit institutions and banks because this type of risk comprises roughly 80% of balance sheet of a bank. Three general types of credit risk models are: structural models, reduced form models and credit rating models. Structural models are based on Merton's general derivative pricing model (Merton, 1974).

**Table 1. An abstract of structural models extensions after Merton model**

Authors (Publication year)	Note
Black-Scholes-Merton (Black and Scholes, 1973, Merton, 1974) (1973 and 1974)	<ul style="list-style-type: none"> <li>• Introduction of European call option pricing base model (standard) to calculate the probability of default.</li> <li>• Solving nonlinear equations for the call option valuation simultaneously (output: volatility, value, default probability)</li> <li>• The applied price in this mode is equal to the nominal value of corporate total debt</li> <li>• It considers 1-year default probability</li> <li>• The company's stock value is a function of time and company's value</li> <li>• Company's volatility is symmetric to company's shares (the result of Ito's lemma).</li> <li>• Put option value is a function of four observable variables (risk-free rate, time to maturity, the price of the underlying asset and the applied price) and another variable (volatility) that should be estimated.</li> </ul>
Vassalou and Xing(Vassalou and Xing 2004) (2004)	<ul style="list-style-type: none"> <li>• The initial value for volatility is considered as: <math display="block">\sigma_A = \frac{\sigma_E V_E}{V_E + B}</math></li> <li>• The volatility of company is estimated by using the historical data of daily returns assets.</li> <li>• They use the simultaneous Black-Scholes equations.</li> </ul>
Hillegeist et al.(Hillegeist, Keating et al., 2004) (2004)	<ul style="list-style-type: none"> <li>• Expansion of the Merton model is known as Hazard Model</li> <li>• Solving nonlinear equations of Black-Scholes simultaneously. They use <math>\mu</math> (percentage change in the company's value over two consecutive terms) to calculate their expected returns instead of r-D</li> <li>• Their approach is called HKCL</li> </ul>
Bharath and Shumway(Bharath and Shumway, 2004) (2004)	<ul style="list-style-type: none"> <li>• Expansion of MKV-Merton model without solving simultaneous equations.</li> <li>• Estimating volatility of debt by using linear relationship with the company's stock volatility.</li> <li>• Calculating the volatility of company by using weighted average volatility of stocks and debt</li> <li>• Their approach is called BhSh</li> </ul>
Campbell et al).Campbell, Hilscher et al., 2008) (2008)	<ul style="list-style-type: none"> <li>• They combined Merton default probability with other related variables of default prediction using risk models.</li> <li>• They found the Merton model probabilities had modest role in prediction ability.</li> </ul>
Bharath and Shumway(Bharath and Shumway, 2008) (2008)	<ul style="list-style-type: none"> <li>• They used the measure of Merton distance to default</li> <li>• They did not use simultaneous equations to estimate default probability</li> <li>• They developed Naive approach for Merton model.</li> <li>• They studied statistical and economic importance of Naive approach with Merton DD measure</li> <li>• Basic model default probability is not a good statistic to predict default.</li> </ul>
Agarwal and Taffler(Agarwal and Taffler, 2008) (2008)	<ul style="list-style-type: none"> <li>• Comparing the predictive ability of HKCL and BhSh approaches by Z score in the UK.</li> <li>• Mapping the Z score to the probability of bankruptcy and comparing to default probability of HKCL methods and BhSh methods (very little difference in prediction accuracy).</li> </ul>
Lee(Lee, 2011) (2011)	<ul style="list-style-type: none"> <li>• He proposed a model for redefining the optimal default barrier in MKV-Merton approach</li> <li>• He changed the default barrier of MKV-Merton approach (the equivalent of short-term debt plus a half of long-term debt).</li> <li>• His idea was that the optimal default barrier must be different from one country to another</li> <li>• He called his model GA-KMV</li> </ul>
Afik et al. (Afik, Arad et al., 2012) (2012)	<ul style="list-style-type: none"> <li>• Sensitivity analysis on parameters of put option pricing model (default barrier the expected return and volatility of assets value of the company).</li> <li>• Accuracy of the model is dependent on the company's assets volatility and the expected return.</li> <li>• They proposed some items to improve the accuracy of the model such as using market returns rather than historical stock returns.</li> </ul>
Doumpos et al. (Doumpos, Niklis et al., 2015) (2015)	<ul style="list-style-type: none"> <li>• Provides multi-criteria classification approach that combines accounting data with a structural default prediction model in order to achieve more accurate predictions</li> <li>• A combination of financial data, structural models and credit ratings in the context of integrated risk management, can increase synergy to assess credit risk.</li> </ul>
Han and Ruihuan Ge(Han and Ge , 2016) (2016)	<ul style="list-style-type: none"> <li>• In practice, we need the use of structural models to estimate the value of the company's shares and this estimate will lead to problems.</li> <li>• To solve this problem they presented a new method based on wavelet theory and called it wavelet structural model</li> <li>• They Found that their mode has more sensitivity and higher accuracy compared to time-series models.</li> </ul>

Authors (Publication year)	Note
Ma and Xu (Ma and Xu, 2016) (2016)	<ul style="list-style-type: none"> <li>In order to describe the unexpected default of the Merton model, Ma and Zu used a new jump diffusion mode instead of geometric Brownian motion.</li> </ul>
Jovan and Ahčan (Jovan and Ahčan, 2017) (2016)	<ul style="list-style-type: none"> <li>One of the assumptions of the Merton model is the normality of stock returns which is not necessarily established and therefore the estimate of default probability might be biased.</li> <li>To solve this problem, they developed the Merton model by following stock returns from Normal - Inverse Gaussian distribution (NIG).</li> <li>Then they developed their approach to estimate the probability of default by EM algorithm.</li> <li>Their results showed that the calculated probability of default is from cross-Merton model but the calculated probability of default is robust with their approach.</li> </ul>

### 3. Methodology

The company's assets value depends on the value of cash flows which will be produced in future. In comparison with shareholders' equity, Debts have higher priority and shareholders receive the remaining value. The main assumption of structural models is that default occurs when the value of the company's assets is not enough for reimbursement of the company's debts.

In the credit risk literature, the distance to default (DD) indicates the number of standard deviations that the expected value of asset at due time (VA) is away from the default point. Thus, it can be written as:

$$DD = \frac{\ln\left(\frac{V_A}{X_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma_A \sqrt{T}} \quad (1)$$

$$P_t = N \left[ \frac{\ln\left(\frac{V_A}{X_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma_A \sqrt{T}} \right] \quad (2)$$

Where

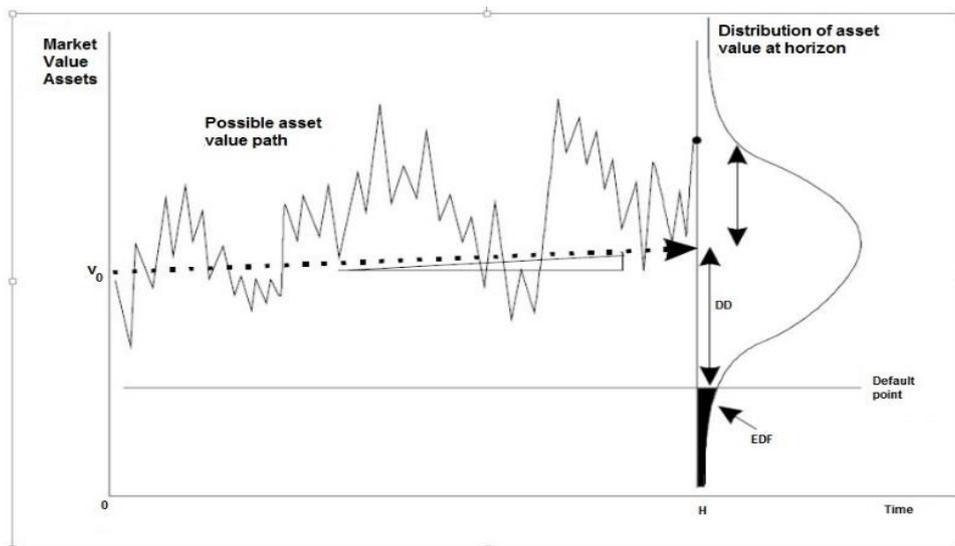
$P_t$  is the probability of default by time  $t$

$V_A t$  is the market value of the firm's assets at time  $t$ , and

$X_t$  is the book value of the firm's liabilities due at time  $t$ .

$\mu$  is the expected return on the firm's asset

$\sigma$  is the standard deviation of assets return



**Fig1. The distribution of probability of default in companies**  
(Bohn and Crosbie, 2003)

Based on the Black-Scholes put option model (BSM), it is assumed that the company's value (V) follows geometric Brownian motion.

$$dV_A = \mu V_A dt + \sigma_A dz \tag{3}$$

Where  $V_A$ ,  $dV_A$  are the firm's asset value and change in asset value,  $\mu$ ,  $\sigma_A$  are the firm's asset value drift rate and volatility, and  $dz$  is a Wiener process.

Finally, Black-Scholes and Merton estimate the company's probability of default by solving the following simultaneous equations system (Black and Scholes, 1973, Merton, 1974).

$$E(V, T) = Ve^{-\delta T} N(d_1) - Be^{-\delta T} N(d_2) \tag{4}$$

$$\sigma_E = N(d_1) \left( \frac{V}{E} \right) \sigma_v \tag{5}$$

Volatility of stock return  $\sigma_E$  can be directly obtained from the historical stock return and the two mentioned simultaneous equations must be solved concurrently in order to obtain numerical estimates for  $V$  and  $\sigma_V$ . (Black and Scholes 1973, Merton 1974)

One of the modifying models of Black-Scholes-Merton which practically has improved this model is known as KMV-Merton approach. KMV-Merton approach uses a new default point for calculating probability of default which is equal to the current debt plus the half of long term debts instead of the total liabilities. In order to accurate the optimal default barrier in KMV-Merton approach, genetic algorithm is used by (Lee, 2011) to redefine it. He found that default barrier could be varied from a country to another. He called his model GA-KMV and defined default barrier as follows:

$$DPT_{GA-KMV} = \alpha * LD + \beta * SD \tag{6}$$

LD and SD are respectively the long term and short term debt. The objective of the proposed model was to estimate optimal coefficients of the long term and short term debt by using genetic algorithm. The values of these coefficients for Taiwan according to his calculation were obtained  $\alpha = 1.9825$  and  $\beta = 1.8813$ .

### 3.1. Empirical Model

Any particular constraint was placed for  $\alpha$  and  $\beta$  in (Lee, 2011). Hence, the estimated default barrier was more than the company's total debt. It was in contrast to the base assumption of Merton model. The idea of KMV model for projection of a new default barrier was completely contrary to this issue because it attempted to define a new default barrier which was less than total liabilities of companies. With this inaccurate estimation, the calculated DD will have a different meaning and the probability of default will not be reliable. When the DPT (default barrier) increases, the distance to default (DD) will be estimated less; hence, the probability of default (PD) will be evaluated more than the real value. Since he used cut off rate in his model to recognize unhealthy companies so with the increase of DPT and PD for companies, more companies would be recognized as unhealthy companies and it would be wrongly concluded that the model would have better performance.

Although the primary idea of his approach is true and logical, there are two points that may cause misunderstanding: First: the long term and short term debts coefficients must be subjected, so that the default barrier between short term and total debts will be satisfied. The goal is to find  $\alpha$  and  $\beta$ , so that DPT, regardless of current and none current debts, will be feasible:

$$\begin{cases} SD \leq DPT \leq SD + LD \\ DPT = \alpha(SD) + \beta(LD) \end{cases}$$

Mentioned constraint has two parts:

$$\begin{cases} \alpha(SD) + \beta(LD) \geq SD \\ \alpha(SD) + \beta(LD) \leq SD + LD \end{cases}$$

Regardless of long debts (LD) and short term (SD) amount, equation (I) will be always true if:

$$\alpha(SD) + \beta(LD) \leq 1 * SD + 1 * LD \rightarrow \begin{cases} \alpha \leq 1 \\ \beta \leq 1 \end{cases}$$

Regardless of long debts (LD) and short term (SD) amount, equation (II) will be always true if:

$$\alpha(SD) + \beta(LD) \geq 1 * SD + 0 * LD \rightarrow \begin{cases} \alpha \geq 1 \\ \beta \geq 0 \end{cases}$$

In order that equation (I) and (II) will be true together, we should intersect between equations (9) and (10), so  $\alpha$  and  $\beta$  will be as below:

$$\begin{cases} \alpha = 1 \\ 0 \leq \beta \leq 1 \end{cases} \quad (11)$$

Default barrier for all companies will be always feasible if the model subjects to equation (11).

Second: new approach will be proposed except for using cut of rate for distinction the distressed companies:

Approach: Suppose in the time horizon, 25 distressed companies and 50 healthy companies are chosen. The best optimization model should be designed:

- 1) To maximize the number of distressed companies which are distinct.
- 2) To minimize the number of healthy companies which are distinct as distressed companies.

With the help of the above proposed method, the mentioned problems will not occur. Model flowchart is designed in Fig2.

## 4. Results

### 4.1. Data description

The used model consists of two groups of financial distressed and healthy companies. The selection criterion of financial distressed companies is the audited companies which included article 141 of the commercial code in 2005-2015. According to this rule, whenever the accumulated loss of the company reaches to the half of its capital, the company will be bankrupted. Accordingly, 25 companies were selected as financial distressed companies and 50 healthy companies were selected as the second group.

The required data for conducting the research were collected through websites and specialized software of the stock market (<http://www.tsetmc.com/>). These data included stock prices, number of stock, increase of capitals, book value of debts, total current debts and total non-current debts of companies in the time horizon.

After collecting the data related to stock companies, organizing and summarizing data was done through Excel. MATLAB software was used to implement the model, calculate the probabilities of

default and implement genetic algorithm. After solving the simultaneous equations, the company's asset value and standard deviation obtained and the genetic algorithm started.

As expected, the result of genetic algorithm was not 0.5 and the algorithm introduced the value of 0.9357 as optimal ratio. By repeating genetic algorithm, it was observed that other solutions near the previous one and exactly with the same fitness function value was reported. It indicates that the asset model had multiple optimal solutions.

**Table 2. The study of the model results for the total model from 2009 to 2014**

Total Companies : 75	
Distressed companies : 25	
The coefficient of long-term debts( $\beta$ )	Identified companies
0.1	10
0.3	11
0.5	11
0.85	14
0.89	14
0.9	15
0.94	15
0.95	14
1	13
2	10

The optimal  $\beta$  was (0.9, 0.94) and for the values of this interval, the model had the best performance.

### 4.2. The study of ability and accuracy of models

Many studies have been conducted during the last years on the selection and validation of credit risk models. Receiver operating characteristic (ROC) curve is one of the most famous methods of model assessment. Receiver operating characteristic curve (ROC) is an assessment criterion for the level of efficiency in classification problems (Engelmann, Hayden et al., 2003).

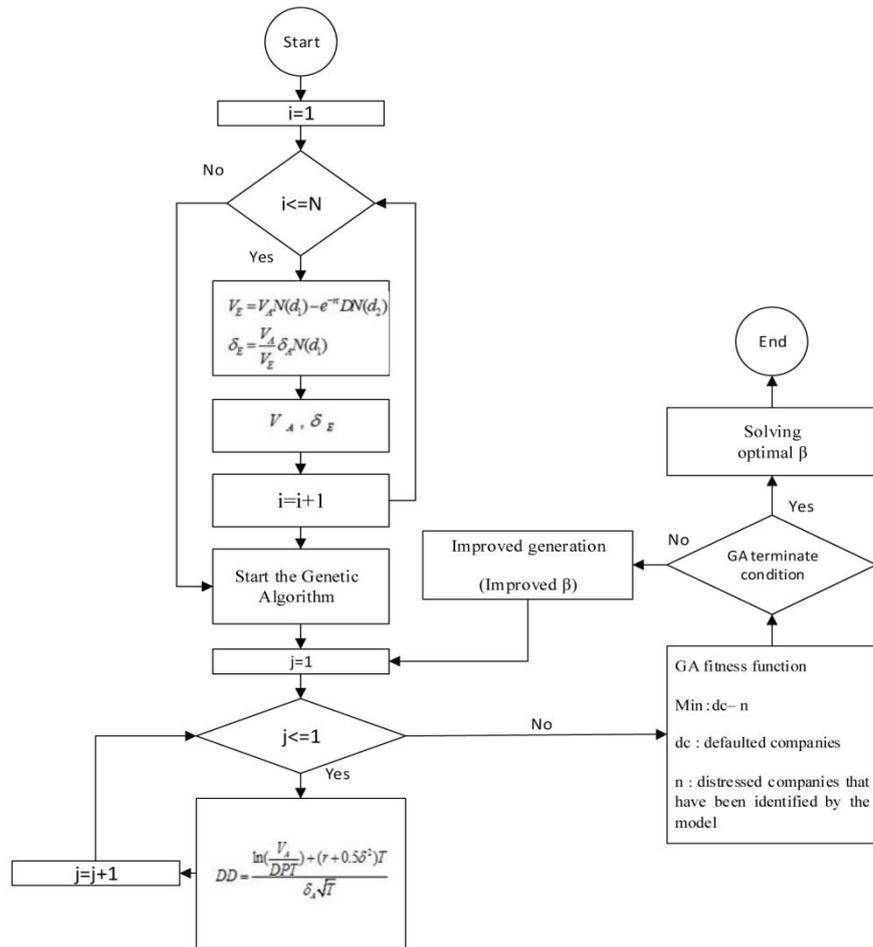


Fig2. Model Flowchart

The ROC curve is a graphic display of sensitivity or correct prediction against wrong prediction in a binary classification system in which separation threshold is variable. Also, this curve is shown by drawing predicted correct positives figure, sensitivity is drawn against (1- specificity). This technique is also used to evaluate different methodologies of credit risk modeling. Sobehert et al(Sobehart and Keenan, 2001) explained Moody’s approach for evaluation of performance and practical considerations of qualitative credit risk models. They showed this method with another name (CAP) and developed other indicators such as accuracy ratio (AR) for evaluation of the model accuracy. They defined the area under ROC curve as the main criterion for evaluating rating models. Engleman et al (Engelmann, Hayden et al.,

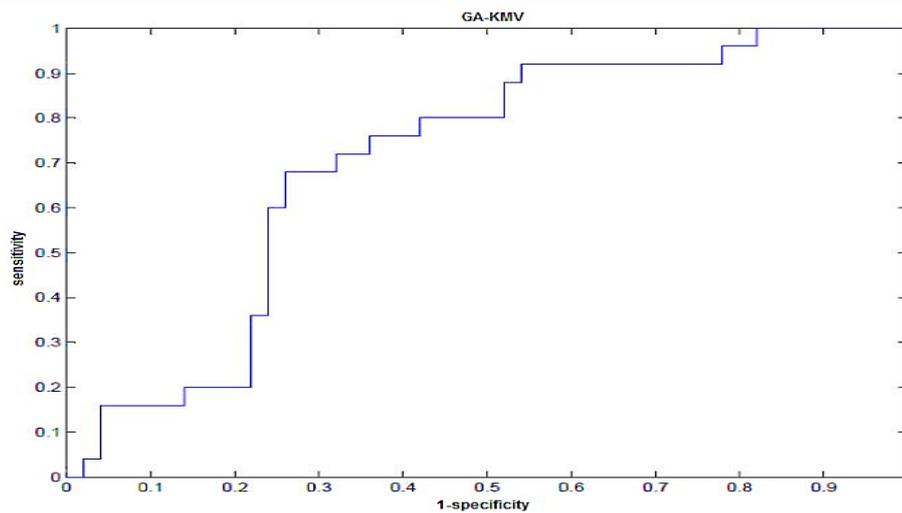
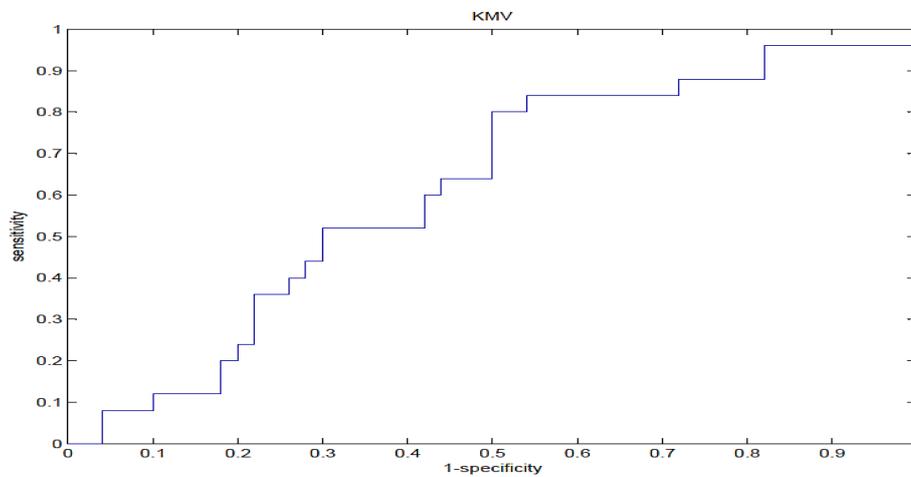
2003) presented a statistical analysis for ROC curve characteristics. They argued that [13]:

$$AR = (2 * AUC) - 1 \quad (12)$$

AUC is the area under ROC curve. For this purpose, in this study the area under ROC curve was calculated through MATLAB.

Table 3. The comparison of the area under the curve and accuracy ratio in different models

Model	AR	AUC
KMV	0.2224	0.6112
GA_KMV	0.4100	0.7050
Random	0	0.500

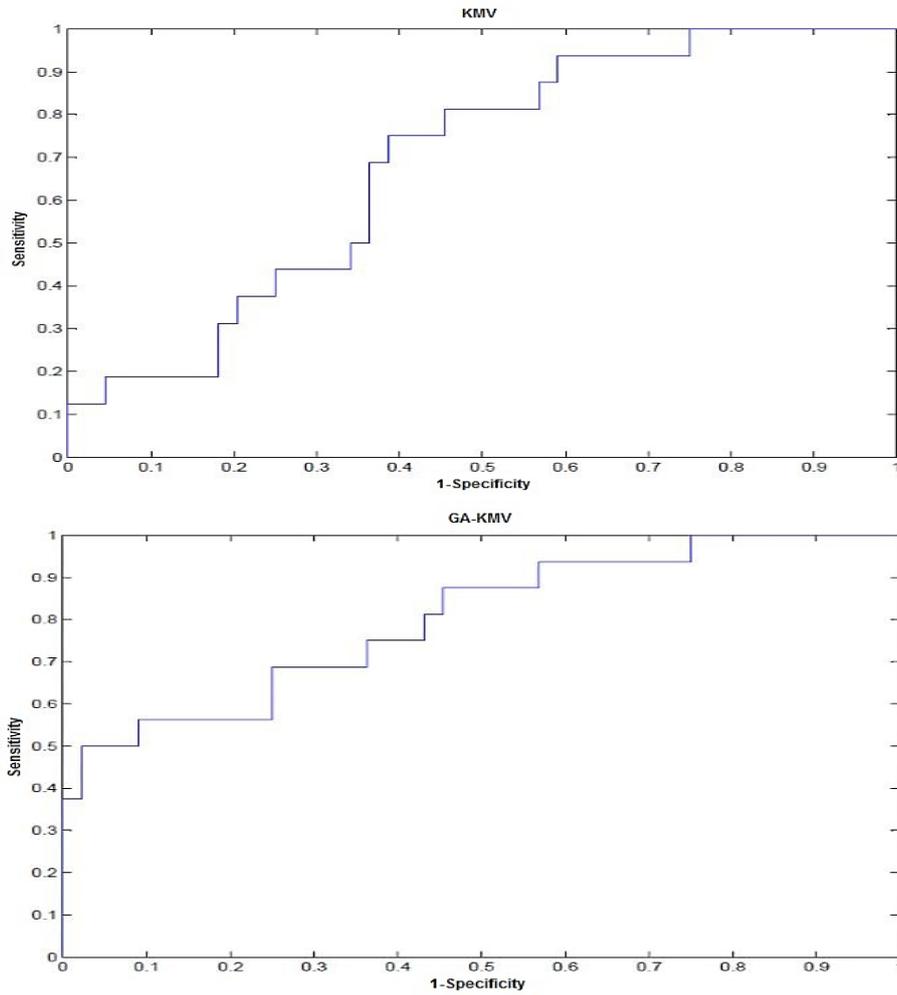


**Figs 5 and 6. The MATLAB software outputs after implementing models**

Although our proposed model showed a better performance in the past in comparison with KMV model, will this model have a good performance in the future prediction? To test this issue and compare their results, we provided the data of 2015 and assumed that it was the beginning of 2015 and then compared the companies identified by the two models at the end of 2015.

**Table 4. Testing the optimal Beta for prediction of bankrupted companies in 2015 and comparing their performance to KMV model**

Total Companies : 60	
Distressed companies : 16	
The coefficient of long-term debts( $\beta$ )	Identified companies
0.1	3
0.3	5
0.5	6
0.85	8
0.9	9
0.94	9



Figs 7 and 8. The MATLAB software output in implementing the model

Table 5. The comparison of the proposed optimal Beta performance and KMV Beta in the model

Model	AR	AUC
KMV	0.3694	0.6847
GA_KMV	0.5994	0.7997
Random	0	0.500

As table 5 shows, the test results indicate that the obtained optimal  $\beta$  in the last section, i.e. interval (0.9, 0.94) has a better performance than KMV model. This result indicates that the proposed model acts better than KMV model both in the past and in the future prediction.

### 4.3. Optimal cutoff rate

Assume that our purpose is not the credit rating of several companies and we intend to make decision on the default of a certain company during the intended time horizon (e.g. one year). In the previous model, after calculating the probability of default of companies, each one that had a less probability was called as the best and each one that had higher probability was called the worst. However, we could not predict which company would or would not lead to default. No opinion can be mentioned about their default, because there was no cutoff rate or reference value for decision-making. We believe that there is an optimal cutoff rate for this decision making that can be

varied from one country to another and from one industry to another and it is better to be calculated separately for each market.

**Calculation method:** we practiced the same companies and data used in the article. We were looking for a cut of rate that the maximum number of correct prediction would happen. Hence, we used genetic algorithm to find the optimal output. Like the previous case, we provided the data of 2009-2014 to calculate optimal cutoff rate and then tested the accuracy and ability of the obtained value for 2015. The figure 0.2265 was obtained as the optimal value for cutoff rate. Then, value was studied for time horizon from 2009 to 2014.

**Table 6. The comparison of distinction ability for different amounts of cutoff rate (2009-2014)**

Cutoff rate	Identified healthy companies(50)	Identified distressed companies (25)	Total Correct identification (75)
0.1	27	22	49
0.2256	37	19	56
0.3	42	7	49
0.5	47	1	48

The comparison of different values for cutoff rate shows that the optimal value obtained by our approach have the best performance. Assume that you have calculated the probability of default of a company and intend to opine about the bankruptcy of this company at the end of the year. You can compare probability of default to the obtained optimal cutoff rate and if probability of default is less than it, you can predict it as healthy company. So if a company distinct as distressed, the probability of true distinction will be 0.74 (37/50). For distinction as healthy company, this probability will be 0.76(19/25) and totally you will predict properly with the possibility of 74.6 in average (56/75). Here, the performance of optimal cutoff rate is studied for 2015.

The possibility of correct prediction associated with being healthy was 86/4% (38/44) and the corresponding figure for being bankruptcy and on average were 62/5% (10/16) and also 80% (48/60) respectively.

**Table 7. The comparison of the distinction ability for different amounts of cutoff rate (2009)**

Cutoff rate	Identified healthy companies(44)	Identified distressed companies (16)	Total Correct identification (60)
0.1	28	13	41
0.2256	38	10	48
0.5	44	0	44

## 5. Discussion and Conclusions

This study attempted to find an optimal ratio for the default barrier of KMV model in order to improve the performance of it. Any specific limitations for  $\alpha$  and  $\beta$  were not considered in (Lee, 2011) study, so the estimated default barrier was more than the company's total debt which was contrary to the basic assumptions of Merton model. The idea of KMV model to create a new default barrier was totally contrary to this issue because it attempted to define a new default barrier which was less than total liabilities of companies. Since he used cut off rate in his model to recognize unhealthy companies so with the increase of DPT and PD for companies, more companies would be recognized as unhealthy companies and it would be wrongly concluded that the model would have better performance.

In this study, new constraints were added to the model in order to resolve mentioned inaccuracies. For this purpose, 25 unhealthy and 50 healthy companies were selected. After implementation of the model, the optimal beta interval (0.9, 0.94) obtained. For the values of this interval, the model had significantly better performance than the KMV model. This was proved by ROC curve according to which the accuracy ratio (AR) of proposed model was 0.4100 whereas that of KMV model was 0.2224. The data of 2015 were used to test the performance of KMV and proposed interval beta. The companies identified by the two models were compared to each other at the end of 2015. The test results showed that the obtained optimal Beta, interval (0.9, 0.94), had better performance than KMV model Beta which is equal to 0.5. As the previous part, this claim was proved with the help of ROC curve based on which the accuracy ratio (AR) of proposed model was 0.5994 while the corresponding figure for KMV model was 0.3694.

At the end, the optimal cutoff rate was calculated and 0.2256 was recognized as an optimal value in order to make decision on bankruptcy or being healthy of a certain company according to its probability of default. The results showed that this value in 2009 to 2014 led to 74/6% correct prediction and for 2015 it led to 80% correct prediction on average.

To enrich this context, various promising directions are recommended. The probability of bankruptcy of companies can be calculated for a time horizon more than one year and search of optimal default barrier. Studying debt structure of companies and their classification according to the relationship between long term and short term debts and calculating optimal default barrier for each class is the other subject. Operating neural networks and fuzzy logic for prediction of the company's assets value and standard deviation must improve prediction accuracy.

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