Credit Risk Predictive Ability of G-ZPP Model Versus V-ZPP Model

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ABSTRACT
Credit risk management is becoming more and more important in recent years. When a company deals with a financial problem, it may not be able to fulfill its financial obligations, which can cause direct and indirect financial losses to shareholders, creditors, investors and other people in the community. Advanced credit risk models that are based on market value include improving credit quality as well as reducing or decreasing credit ratings. In the current study, we investigate a new model called ZPP that was introduced in 2007. This model is one of the advanced models of credit risk and the standard deviation of the model is calculated the GARCH model.

In this survey we test the accuracy of the ZPP model with GARCH and Simple Standard Deviation. In order to test the accuracy of the model, we have chosen two models: firms with financial problems and companies with financial health, and in each group, we estimated the probability of default by two models and then compared the probability of default with each other. Finally, we found the predictive ability of the G-ZPP model which was obtained by the GARCH model was better than the Variance-ZPP model.

Keywords
ZPP, Credit Risk, Probability of default, Monte-Carlo simulation

1. Introduction

The credit risk issues and methods for identifying and predicting risks have been steadily evolving over the past few decades.(Elizalde, 2005) When a company deals with a financial problem, it may not be able to fulfill its financial obligations, which can cause direct and indirect financial losses to shareholders, creditors, investors and other people in the community.

Credit risk can create a financial crisis that is a systemic risk for the global economy(Crouhy, Mark, & Galai, 2010). A number of researchers have devoted much effort to solve this community need by proposing different models. the model of Altman (1968) and Merton (1974) can be named among these models.

The traditional models estimate the default probability (PD) rather than the losses related to default event (LGD: loss given default). These models don't consider the downgrades and upgrades in credit quality that are studied by market models, but they analyze the "failure" like the bankruptcy, the default or liquidation(Falavigna, 2006) In addition, asset volatility is ignored in accounting-based default prediction models. Whereas advanced models are based on market value and they consider the downgrades and upgrades in credit quality. Over the last few years, new models for credit risk modeling have been introduced such as structural models and reduced form approach or intensity-based model.

The structural model or the contingent-claims approach provides an alternative to the credit rating changes in credit portfolio modeling. In this model, the economic value of default is considered as a put option on the value of the company's assets.

The Merton Model was introduced in 1974 and it is an application of the Black Scholes (1973) formula to the pricing of debt. This model is based on the asset value model, studied by Merton. The approach proposes the default process endogenous and related to the capital structure of the firm. When the value of the assets of a firm goes down a given critical level, the default happens(Kabir, Worthington, & Gupta, 2015). When the asset value falls behind the liabilities value, there is no value of holding equity. If the value of the equity is zero or negative, the firm cannot fully cover creditors' claims. As a result, the firm is in default.

Although the Merton model was developed in many ways, some drawbacks to the Merton model were still maintained. For example, the default assumption only occurs when the debt matures. It was an unrealistic hypothesis in the first category of structural models. In the real world, a company cannot pay its bills and other obligations at any time during of the time of debt issue and the time of the bond maturity. Another unrealistic assumption in the first generation of structural models is fixed-rate. The fixed interest rates cannot be hold in reality. As a result, a stochastic process of the interest rates has been introduced many researchers (X. Zhang, 2017).

The KMV model is one of the models that is the base on structural models. KMV methodology is introduced by Kealhofer, McQuown and Vasicek, which is bought by Moody's in 2002. The KMV-Merton model assumes that as long as the value of a firm's assets covers the book value of its debt, the company will not default. In other words, the KMV model states that at the point of default, the market value and the book value of the assets are combined and many transactions and services continue to generate debt(Lee, 2011). In addition, this model proposed that long-term debts created a respiratory time for the company, so their effects on the company's default is less than current liabilities.

However, the KMV model does not assume that asset values are normally distributed and does, therefore, not use the cumulative normal distribution to convert distance to default into default probabilities. Instead the KMV model uses its large historical database, which contains data on historical defaults and bankruptcy frequencies, to obtain a relationship between distance to default and default probabilities(Göransson & Grétarsson, 2008).

KMV methodology solved drawbacks of Merton model but some drawbacks remained in it. KMV approach uses data from reports or financial statements which, in the case of less developed countries, probably are even less reliable. So the ZPP model was introduced by De Giuli, Fantazzini, and Maggi (2007), which stands for Zero Price Probability(Fantazzini, Giuli, & Maggi, 2007). This method also measures the credit risk of companies. The main focus is on the new simulation-based approach rather than the older established models. The comparison is done to get an implicit idea of the power of both models. This was done by calculating the Zero-Price Probability using Monte Carlo simulations and also calculating the KMV, both models are presented for 500 observations and on a forecast horizon of one year. They are presented for several firms, both defaulted and good financial health, from different industries and regions.

They said however the stock-market is usually assumed to be near efficient for at least the more developed economies in the world. On less developed capital markets, the ZPP approach might not be the best one, but considering that the KMV approach uses stock prices as well and data from reports which, in the case of less developed countries, probably are even less reliable, this comparative drawback of the ZPP isn't that important.

This model means that the probability of the next trading day is equal to today. According to empirical studies of these researchers, the main advantage of the ZPP model over the KMV model is that it avoids the issue of asset visibility and considers the company's asset value as a probable claim on stock prices and securities.

In the ZPP method, researchers used the GARCH model to obtain price probability. Here the GARCH-ZPP is called G-ZPP and simple standard deviation is called V-ZPP and we want to calculate the probability of default through two method and compared the prediction accuracy.

Therefore, the present study is based on two sample groups of companies which are listed in Tehran Stock Exchange and it has been investigated through two samples whether G-ZPP can be better than V-ZPP model or not. And we wanted to know which model is best to discriminate two groups.

2. Literature Review

Many statistical methods have been applied in credit risk analysis, such as discriminant analysis (DA), regression methods and logistic regression (LR) models. Durand (1941) makes the interesting remark that in practice it is difficult to make a precise distinction between *good* and *bad* loans. Durand (1941) first shows that DA can predict credit repayment with fair accuracy(Durand, 2008). Altman (1968) introduces the Z-score model to predict the firms' credit risk based on accounting information(Altman, 2000).

Since the1990s, AI methods have become increasingly popular for investigating financial credit risk. Compared to statistical methods, AI methods do not have to assume a population distribution and thus can be applied to a broader range of contexts, especially when the relationship between variables is potentially non-linear. In general, the flexibility of AI methods grants them better performance than statistical methods in credit risk prediction, but their better performance is not conclusive(Li, Yang, & Zou, 2016).

In his pioneering work, Merton (1974) assumes that the firm value follows a geometric Brownian motion and then constructs a structural model that assesses firms' credit risk. Although theoretically appealing and intuitive, Merton's model is subject to limitations in its assumptions including the observability and tradability of the firm value, log normal distribution of asset value and fixed maturity of debts(Merton, 1974).

Later KMV Corporation relaxes some of the assumptions of Merton's model and brings it to industry applications. Some researchers have compared the performance of KMV and competing models.

Bohn (2000) compares the credit risk assessments of S&P and Moody's and the expected default frequency (EDF) of the KMV model(Bohen, 2000). In addition, much research has been done on the KMV methodology. Crosbie and Bohn (2002) released that the relationship between asset volatility and equity volatility and they justified the assumption of KMV model(Crosbie, P. J., 2002). Bharath and Shumway(2008) investigated the accuracy of the KMV-Merton in predicting default by formulating its naïve alternative probability (Bharath, S. T., & Shumway, 2008). Their results show that the naïve predictor performs slightly better than both the KMV model and a reduced-form model.

Charitou et al. (2013) extend the Black–Scholes–Merton bankruptcy model by estimating volatility directly from market-observable returns on firm value. Their empirical results show that parsimonious models actually perform better than the alternative, more sophisticated, models(Charitou, Dionysiou, Lambertides, & Trigeorgis, 2013). Câmara, Popova, and Simkins (2012) modify Merton's (1976) ruin option pricing model by extending the geometric random walk to the delta-geometric random walk and show that the probabilities of default estimated from the modified model are equal or superior to other credit risk measures studied based on cumulative accuracy profile (CAP) and ROC(Câmara, Popova, & Simkins, 2012). Gordy and Marrone (2012) apply granularity adjustment methodology to Credit Metrics and KMV Portfolio Manager. Zhang and Shi (2016) apply Particle swarm optimization is used to improve the KMV model and in their study Particle swarm optimization outperforms genetic algorithm.(Y. Zhang & Shi, 2016).

The ZPP model was introduced by De-Giuli, Fantazzini, and Maggi (2007) who have overcome the KMV model's drawback of the non-observability of assets' value and volatility. The market value of the assets of a company can be deemed as a claim on its traded securities: stocks and bonds. Using copula theory, their paper extends a methodology to extract default probabilities from stock prices. The next section will provide a brief description of the ZPP model.

De Giuli, Fantazzini, and Maggi (2007) propose, which can measure the credit risk of corporations. The main advantage of ZPP model over KMV model is that it avoids the asset observability issue, and it treats a firm's asset value as a contingent claim of its stock and bond prices. It can measure and detect credit risk of corporations and can distinguish between financially challenged firms and financially healthy firms better than the KMV model.

In recent years, there have been studies on credit risk in the Chinese markets with various methodologies. Chen, Wang, and Wu (2010) modify the original KMV model with tunable parameters to measure the credit risk of Chinese listed SMEs. Zhang, He, and Zhou (2013) analyses the 187 high tech listed companies' credit risk in China using Cox model and find that the independent innovation capacity can reduce credit risk of these high-tech enterprises. Wang and Ma (2011) propose an integrated ensemble approach to predict corporate credit risk, RS-Boosting, which is based on boosting and random subspace. Lili Li, Jun Yang & Xin Zou (2016) studied on companies listed on the China Stock Exchange. The researchers also examined and validated the performance of the ZPP model(Li et al., 2016). There is no research on the ZPP model in Iran.

3. Methodology

3.1. ZPP model

the ZPP model simplifies the asset value function with copula theory and simulates it with the Monte Carlo method. This model can identify and measure corporate credit risk. The basic idea of the ZPP model is as follows. The final or ultimate value of the firm at the maturity of its debt is

$$A_t = G(E_T, B_T; T) = max(E_T, B_T, 0) I_{[(E_T \geq 0),(0 \leq B_T \leq D)]} \quad \textit{3-1}$$

where B_T is the bond value, E_T is the equity value, D represents the debt and I is the indication function. In other words, I is a function that if one of the conditions listed below is true the value is one, otherwise the value is zero.

Under the assumption of complete markets Equation (3-1) can be written in a continuous form:

$$\boldsymbol{A_t = G(E_t, B_t; t) = P(t,T) \int_0^\infty \int_0^\infty G(E_T, B_T; T) q(E_T, B_T | F_T)\, dE_T dB_T} \quad \textit{3-2}$$

where P(t, T) is the risk-free discount factor, $q(E_t, B_T|F_T)$ is the risk-neutral probability density function for the bivariate claim $G(E_T,B_T;T)$.

The above equations are difficult to solve, especially when the market is not efficient. Therefore, in order to solve this problem, the theory of Coppola is used. The above equation can be written as follows:

$$\boldsymbol{A_t = P(t,T) \int_0^\infty \int_0^\infty G(E_T, B_T; T)\, C_{E,B}(F_E, F_B | F_t) f_E(E|F_t)} \quad 3\text{-}3$$

where $C_{E,B}$ is the bivariate copula density between bond and stock prices. F_E and F_B are the marginal densities of the above model. Because of the lack of liquidity in corporate bond markets, it is nearly impossible to obtain the risk-neutral marginal density function of in distribution of bond and stock prices. So we use the asssumption of the risky interest rate:

$$A'_t = P_r(t,T) \int_0^\infty \int_0^\infty G(E_T, B_T(i_T); T) c_{E,i}(F_E, F_i | F_T)\ F_E(E \quad 3\text{-}4\text{-}$$

Considering that BT and Pr(t, T) are known at time t, so we need the stock price distribution only. It means we don't know (ET):

$$A''_t = P_i(t,T) \int G(E_T, B_T; T)\, f_E(E|F_T) dE_T \quad \textit{3-5}$$

Thus the firm value can be simulated with Monte Carlo methods:

$$\boldsymbol{A^{\sim}_t = P_i(t,T)\frac{1}{N}\sum_{i=1}^{N} G\left(E^{\sim}_{i,T}, B_T; T\right)} \tag{3-6}$$

$$\boldsymbol{= P_i(t,T)\frac{1}{N}\sum_{i=1}^{N} max\left(E^{\sim}_{i,T}, B_T, 0\right)_{[(E_T \geq 0),(0 \leq B_T \leq D)]}}$$

When a company's value is smaller than its debt, it cannot repay its liabilities, and its equity value is below zero. If we allow the domain of equity price to be (-∞,∞), the default probability can be retrieved by $Pr[E_T \leq 0]$ or $Pr[P_T \leq 0]$, where P_T is the stock price. This is the origin or base of ZPP model.

This approach has several advantages over the Merton-type structural models. For instance, this model does not need the firm's volatility, which has proven to be quite complicated to calculate. This approach does not use a log-normal distribution like almost every structural model. This method can estimate the default probability for any given time horizon, while the Merton-type models have been shown to give an almost zero probability of default when the maturity of debt is nearing its end(Göransson & Grétarsson, 2008). There are four steps to perform ZPP simulation:

- Calculate the conditional variance
- Calculate one price trajectory using the conditional variance
- Repeat step two for a set amount of times, in this case 10000
- If historical values are to be done repeat the three above for each time adapting the information for each time.

ZPP model uses the price changes to calculate Monte Carlo simulation instead of today's price(Lento & Gradojevic, 2013). Because of the application of price changes allows to produce the negative values, and on the other hand, the use of changes in the Monte Carlo simulation causes the predicted data tolerance to be reduced or, in other words, to produce less outliers(Trueck & T.Rachev, 2009).

$$\boldsymbol{X_t = P_t - P_{t-1}} \tag{3-7}$$

In the initial approach of ZPP is used GARCH model with AR(3) in Monte Carlo simulation, which is named G-ZPP. It should be noted that the variance is used in V-ZPP which is calculated from the price changes again and not from the logarithm return of the price.

Finally as we said before the probability of default is calculated by a simple ratio:

$$\boldsymbol{PD = n/N} \tag{3-8}$$

The default probability is simply the number of times n out of N when the price touched or crossed the barrier along the simulated trajectory. In other words, the number of times the price has fallen below zero or negative.

It should be noted that all of the above calculations are done in R software because the simulation in Excel is not possible for this number of data. In this research, after entring previous prices in R software, which were being traded the last 248 days days, we use the formula to calculate GARCH and then calculate G-ZPP.

$$\boldsymbol{\delta_E^2 = \frac{\sum_{i=1}^{n}(R_t - \bar{R})^2}{n-1}} \tag{3-9}$$

Due to calculate the equity volatility in Excel we used the following formula:

$$\boldsymbol{\sigma_E = STDEV.S\sqrt{n}} \tag{3-10}$$

n: The number of price (248 days)

STDEV.S :The standard deviation of the prices

4. Results

After sampling the listed companies and selecting the sample that meets our requirements for this research, we extract the trading price data to calculate the probability of default through the above two models.

Requirements for selecting the firms with financial problems as follows:

- Debt-to-Equity ratio more than one;
- Trading over 2 million shares during of a trading 90-day period;
- With negative ROE.

Above requirements give us the sample of 26 firms with financial problems, which are mainly under the cement and construction industry, some of which are part of the financial services sector. But we have to exclude one of them from our sample which is named Ghavamin bank because it is not traded and it is excluded or exited from capital market.

On the other hand, the requirements for selecting the companies with financial health as follows:

- Debt-to-Equity ratio less than one;
- Trading over 5million shares during of a trading 90-day period;
- With positive ROE;
- Companies have grown more than 10 percent during the last five years.

The strict requirements are in order to choose the best sample of the most ideal companies in the group of financial-health firms. The strict requirements give us 26 companies. It is necessary to reduce the number of healthy samples by eliminating one company because we need two samples which are equal. Eventually both samples were created with 25 companies.

Then, both models will be tested on both groups and it will be discussed which of the above models has better predictive power to recognize and differentiate between two groups of companies?!

In the following table, as mentioned above, companies are divided into two categories:

- Group (One): Companies with Financial Health
- Group (two): Companies with Financial Problems

Table 1: Default Probabilities in the G-ZPP Model

G-ZPP	Volatility of asset	Asset value (million riyal)	Group(Two)	G-ZPP	Volatility of asset	Asset value (million riyal)	Group(One)
0.1097	0.4026	2,017,500	1	0.0044	0.2385	175,440,000	1
0.1904	0.3268	4,200,110	2	0.0003	0.3869	11,367,000	2
0.1948	0.2786	5,619,452	3	0.0001	0.2564	57,460,000	3
0.1012	0.0113	74,525,438	4	0.0000	0.2482	221,892,000	4
0.0640	0.2528	9,522,000	5	0.0721	0.4599	4,376,490	5
0.3346	0.3554	2,686,500	6	0.0047	0.2177	517,920,000	6
0.4166	0.3682	1,360,435	7	0.0000	0.3246	168,091,300	7
0.1989	0.4630	5,436,600	8	0.0270	0.2610	326,508,000	8
0.2764	0.2940	3,552,256	9	0.0001	0.3914	7,961,250	9
0.5055	0.5582	12,728,116	10	0.0000	0.2766	213,500,000	10
0.2569	0.3371	5,744,200	11	0.0100	0.3275	9,739,600	11
0.1281	0.1566	6,934,485	12	0.0636	0.4482	15,633,200	12
0.6124	0.9095	10,133,688	13	0.0000	0.2862	220,836,000	13
0.0963	0.1047	1,027,200	14	0.0036	0.4497	330,912,000	14
0.1557	0.5098	799,534	15	0.0001	0.3512	14,843,400	15
0.1573	0.4256	13,186,250	16	0.0000	0.3518	60,963,500	16
0.0949	0.2496	6,029,607	17	0.0004	0.2490	57,617,280	17
0.0840	0.3658	1,909,218	18	0.0002	0.4599	3,011,250	18
0.1645	0.0390	55,163,331	19	0.0084	0.7176	19,485,138	19
0.2401	0.2037	2,233,500	20	0.0004	0.3907	14,604,000	20
0.6353	1.0816	5,039,967	21	0.0044	0.4129	15,680,800	21
0.0193	0.5733	3,883,800	22	0.0006	0.3870	5,527,480	22
0.4671	0.7410	14,685,308	23	0.0002	0.4965	2,099,989	23
0.2470	0.3791	7,391,102	24	0.0811	0.6103	2,956,800	24
0.3937	0.3091	710,446	25	0.0037	0.3096	19,423,600	25

Table 2: Default Probabilities in the V-ZPP Model

V-ZPP	Volatility of asset	Asset value (million riyal)	Group(Two)	V-ZPP	Volatility of asset	Asset value (million riyal)	Group(One)
0.0941	0.4026	2,017,500	1	0.0002	0.2385	175,440,000	1
0.0743	0.3268	4,200,110	2	0.0001	0.3869	11,367,000	2
0.0176	0.2786	5,619,452	3	0.0000	0.2564	57,460,000	3
0.0001	0.0113	74,525,438	4	0.0000	0.2482	221,892,000	4
0.0056	0.2528	9,522,000	5	0.0001	0.4599	4,376,490	5
0.0043	0.3554	2,686,500	6	0.0000	0.2177	517,920,000	6
0.0320	0.3682	1,360,435	7	0.0000	0.3246	168,091,300	7
0.0654	0.4630	5,436,600	8	0.0000	0.2610	326,508,000	8
0.0043	0.2940	3,552,256	9	0.0000	0.3914	7,961,250	9
0.0764	0.5582	12,728,116	10	0.0000	0.2766	213,500,000	10
0.0564	0.3371	5,744,200	11	0.0000	0.3275	9,739,600	11
0.0017	0.1566	6,934,485	12	0.0001	0.4482	15,633,200	12
0.3590	0.9095	10,133,688	13	0.0000	0.2862	220,836,000	13
0.0020	0.1047	1,027,200	14	0.0000	0.4497	330,912,000	14
0.0032	0.5098	799,534	15	0.0000	0.3512	14,843,400	15
0.0521	0.4256	13,186,250	16	0.0000	0.3518	60,963,500	16
0.0210	0.2496	6,029,607	17	0.0000	0.2490	57,617,280	17
0.0129	0.3658	1,909,218	18	0.0000	0.4599	3,011,250	18
0.0000	0.0390	55,163,331	19	0.0000	0.7176	19,485,138	19
0.0119	0.2037	2,233,500	20	0.0001	0.3907	14,604,000	20
0.1968	1.0816	5,039,967	21	0.0000	0.4129	15,680,800	21
0.0194	0.5733	3,883,800	22	0.0000	0.3870	5,527,480	22
0.3210	0.7410	14,685,308	23	0.0035	0.4965	2,099,989	23
0.1314	0.3791	7,391,102	24	0.0250	0.6103	2,956,800	24
0.1146	0.3091	710,446	25	0.0021	0.3096	19,423,600	25

We avoid to release the name of companies because of confidentiality but we publish the asset value of them.

As we can see in the above tables, in the financial health group through the V-ZPP model, the probability of default is almost zero but G- ZPP model could predict the probability of default in Group(two) better than the V-ZPP model. Therefore we can see less errors and better predictive power in G-ZPP model.

In sum, it can be argued that the accuracy of the G-ZPP model is more accurate in distinguishing between groups of firms with financial problems and those with financial health.

5. Discussion and Conclusions

The main purpose of this paper has been to investigate the discriminatory power of the G-ZPP model compared to V-ZPP. The ZPP models are based on copula theory and predict credit risk with Monte Carlo simulations. This was done by comparing the calculations of default probabilities for a number of firms varying from default.

The prediction results of these models were compared with each other. Most firms that are in “good” financial health get low default probabilities for all estimates in both models. But most of them that are in “distressed” financial firms get high default probabilities which are estimated in G-ZPP.

The comparison shows that the G-ZPP model performs better than the V-ZPP model in the Iranian capital market. The efficiency of the V-ZPP model in finding companies with a high probability of bankruptcy is less accurate than the G-ZPP model, so the G-ZPP model can more accurately distinguish high credit risk companies between low credit risk companies.

The results of V-ZPP model are not as clear-cut as for the G-ZPP model therefore we understand the result of

ZPP model tends to vary significantly. Thus, it gives us an idea that one important factor is the volatility estimation which is required for the ZPP estimation.

Eventually, the predictions of both models were not accurate 100 percent. It is still possible to improve ZPP model and the type of estimation of stock fluctuations. Therefore, one of the future investigations that can be done in this field, is that researchers investigate the effect of different methods of estimating the share-price fluctuations on ZPP model. It is also suggested that past data can be used to calibrate ZPP predictions by using techniques such as machine learning.

References

1) Altman, E. (2000). Predicting Financial Distress Of Companies: Revisiting The Z-Score And Zeta. Handbook of Research Methods and Applications in Empirical Finance, 5.
2) Bharath, S. T., Shumway, T. (2008). Forecasting default with the Merton distance to default model. Review of Financial Studies, 21, 1339–1369.
3) Bohen, J. (2000). Survey of contingent claims-Approaches to risky debt valuation. Working Paper.
4) Câmara, A., Popova, I., Simkins, B. (2012). A comparative study of the probability of default for global financial firms. Journal of Banking & Finance, 36(3), 717–732.
5) Charitou, A., Dionysiou, D., Lambertides, N., & Trigeorgis, L. (2013). Alternative bankruptcy prediction models using option-pricing theory. Journal of Banking & Finance, 37(7), 2329–2341.
6) Crosbie, P. J., B. (2002). Modeling default risk. Research Paper, Moody's KMV.
7) Crouhy, M., Mark, R., Galai, D. (2010). Risk Management (2nd editio). McGraw-Hill Professional.
8) Durand, D. (2008). Risk Elements in Consumer Instalment Financing.
9) Elizalde, A. (2005). Credit Risk Models II: Structural Models.
10) Falavigna, G. (2006). Models for Default Risk Analysis Focus on Artificial Neural Networks, Model Comparisons, Hybrid Frameworks. Ceris-Cnr and University of Bergamo.
11) Fantazzini, D., Giuli, M. E. De, Maggi, M. (2007). A New Approach for Firm Value and Default Probability Estimation beyond Merton Models. Moscow School of Economics, 23.
12) Göransson, P., Grétarsson, Ì. A. (2008). A Comparative Study between the KMV and the Zero-Price Probability for Default Prediction. University of Lund.
13) Kabir, M. N., Worthington, A., & Gupta, R. (2015). Comparative credit risk in Islamic and conventional bank. Pacific-Basin Finance Journal, 34, 327–353.
14) Kealhofer, S., Kurbat, M. (2000). Benchmarking quantitative default risk models:A validation methodology. Research Paper, Moody's KMV.
15) Lee, W. (2011). Redefinition of the KMV model's optimal default point based on genetic algorithms-Evidence from Taiwan. Expert Systems with Applications.
16) Lento, C., Gradojevic, N. (2013). The Effectiveness of Option Pricing Models During Financial Crises. In Rethinking Valuation and Pricing Models.
17) Li, L., Yang, J., Zou, X. (2016). A study of credit risk of Chconflict of interest was reported by the authorsinese listed companies: ZPP versus KMV. Economics, 48(29).
18) Merton, R. (1974). Theory of Rational Option Pricing. Bell Journal of Economics and Management Science, 4 (1), 141–183.
19) Trueck, S., T.Rachev, S. (2009). Rating Based Modeling of Credit Risk. Elsevier.
20) Zhang, X. (2017). Essays in Credit Risk Management. University of Glasgow.
21) Zhang, Y., Shi, B. (2016). Non-tradable shares pricing and optimal default point based on hybrid KMV models: Evidence from China. Knowledge-Based Systems, 110, 202–209.