Investigating the Impact of Time-varying Volatility of Macroeconomic Indices on the Predictability of Optimal Stock Portfolio Return in Tehran Stock Exchange

Hosein Maghsoud  
PhD Candidate  
Department of Financial Management, Science and Research Branch, Islamic Azad University  
Tehran, Iran  
maghsod@gmail.com

Fraydoon Rahnamay Roodposhti  
Department of Financial Management, Science and Research Branch, Islamic Azad University  
Tehran, Iran  
(Respondent Author)  
rahnama.roodposhti@gmail.com

Hamidreza Vakilifard  
Department of Accounting, Science and Research Branch, Islamic Azad University  
Tehran, Iran

Taghi Torabi  
Department of Economy, Science and Research Branch, Islamic Azad University  
Tehran, Iran  
taghi.torabi@yahoo.com

ABSTRACT
In this study, 3 models of Time-Varying Parameters (TVP), Dynamic Model Selection (DMS) and Dynamic Model Averaging (DMA) and a comparison with the Ordinary Least Squares (OLS) method in MATLAB in the time period 2003-2013 (with data on a monthly basis) are discussed. In the present study, the variables of unofficial exchange rate changes, interest rate changes and inflation in oil price forecast returns for stocks in Tehran Stock Exchange are used. The study concludes that dynamic models with time-varying parameters are more accurate in predicting returns in the Stock Exchange, in a way that the MAFE and MSFE models, DMA, DMS which have complete dynamics are more efficient than other models. As a consequence, it can be said that the variability of the coefficients of the variables in the TVP model cannot lead to higher accuracy in predicting returns in the Stock Exchange, and it is required that the dynamics of time-varying variables of the model used to predict stock returns be taken into consideration.

Keywords: Macro indexes, optimal portfolio, stock returns, time-varying parameter method, dynamic models
1. Introduction

One of the problems of investors in adopting expected return prediction models is that these models are seriously sensitive to different markets and conditions and lack sufficient stability. According to previous studies, although there might be evidence for the predictability of expected return prediction models, such predictions are weak to an extent that investors do not use them in practice. A set of rationales support this conjecture that standard approaches may show a weak performance out of samples. The first one is that regression model does not take the very important properties of stock return into account. Especially, constant volatility assumption seriously disagrees with observed data because return volatility changes over time. According to a study by Johannes, et al. (2014), ignoring such volatility creates suboptimal portfolios which are merely based on the expected return with constant variables over time and this is why they show a poor performance.

In addition, linear regression model assumes that the relationship between $x_t$ and $x_{t+1}$ is time-invariant. Theoretically, certain asset pricing models, such as Menzly, Santos, and Veronesi (2004), or Santos and Veronesi (2006) imply a relationship between the equity premium and a time-varying relation between stock premium and $x_t$ time-varies. From practical point of view, Paye and Timmerman (2006), Lettau and Nieuwerburgh (2008), Henkel, Martin and Nardari (2011) and Dangel and Halling (2012) found evidence for a time-varying relation between stock return and conventional predictors. The goal of this paper is to introduce extensions to deal with these features and reevaluate the out-of-sample performance.

This study is important because it explores causes and stabilizes the prediction capability of pricing models through assuming more realistic assumptions. This study addresses the problem of an investor who is interested in adjusting an optimal portfolio and acquires investment information over time. In order to build an accurate model, it is necessary to take all important properties into account such as predictable expected returns, time-varying volatility and parameter uncertainty. The aim of such investigations is to discover how sequential learning of investors about parameters, state variables and models change when new data are introduced (change of macro indices).

2. Literature Review

Economic and financial phenomena are characterized with dynamism and time-varying phenomena. Ignoring dynamism leads to the excessive simplification of the phenomena, therefore, those models grounded on such over simplifications will be generally unrealistic and lead to misinterpretation of phenomena. The trade-off between risk and expected return is an essential principle of financial theory. Expected return may vary over time by the change of risk factors. Such a time-varying change in the expected stock return disturbs the random walk of prices. Therefore, the majority of financial experts believe that it is impossible to evaluate whether or not stock prices are predictable without taking relevant risks into account (Pesaran and Timmerman, 1995). According to Stock and Watson’s opinion (2008), one of the important problems of traditional models was their failure in giving an accurate prediction over time: so that some models well estimated in prosperity situation and some others well estimated in depression situation. This resulted in the emergence of time-varying parameters as well as Monte Carlo Markov Chain (MCMC) models which could create great models (models with numerous parameters) over time. In such models, estimation coefficients can vary over time. It has been observed that older models failed to calculate parameters in such conditions due to the variation of condition, structural breaks and cyclic changes (Koop and Kroubils, 2011).

In recent decades, scholars and researchers have suggested different models for predicting stock return. Capital Asset Pricing Model (CAPM) is one of the most well-known models, which has maintained its special place until now (Sharpe, Lintner and Mossin, 1965). As an extension of Markowitz portfolio theory, this model claims that the risk of security market alone can explain the volatility of security return by itself. Despite many advantages, this model has been criticized by researchers. The most important disadvantage of this model is that it attributes all volatilities to the market return which is not a realistic assumption. Fama and Makbeth (1992) rejected this claim and claimed that unlike Sharp's claim, the beta value of market cannot completely explain stock risk. On this ground, Fama and French (1993) introduced the so-called Tree Factor Model. This model uses Ordinary Least Squares (OLS) regression and tries to eliminate CAPM inability in explaining expected return.
return by adding size and Book Value to Market Value Ratio (BV/MV). Despite many challenges, it was successful in different markets, therefore, it can be argued that it is currently employed by the players of capital market as a developed tool for predicting expected return, measuring capital cost and assessing portfolio performance.

The main disadvantage of tree factor model is that Fama and French have assumed constant beta coefficients over time in their OLS-based regression model; while the studies of other researchers such as Blume (1971), Lvey (1971), Rosenberg (1985), and Ferson and Harvey (1991) on the reputable stock exchange markets of the world show that beta coefficients are not constant over time.

Adrian and Fronzoni (2005) have adopted the concept of learning in the estimation of beta coefficients. They believe that the traditional OLS ignores previous errors in training investors and this may result in CAPM failure in practice. Their model measures the effect of long-term learning, as a non-observable variable, on beta coefficients through state-space model. Assuming non-constant beta coefficients, Huang and Hueng (2007) used state-space model and examined conditional CAPM model. They found that there is a positive and a negative relation between risk and return in prosperity and depression conditions, respectively. Das et al. (2010) estimated CAPM beta coefficients using Kalman filter. They found that the estimation of beta coefficients using Kalman filter promotes the accuracy of this model in forecasting return. Nieto et al. (2014) compared OLS, GARCH and Kalman filter in Mexico stock exchange and found that Kalman filter shows a better performance in estimating beta coefficients compared with other techniques. In a PhD thesis, Fux et al. (2014) studied return predictability and structure modeling. According to their findings, the investor can increase its idealistic level up to 1.2%, compared with OLS-based predictions, using dynamic averaging models where instabilities, time-varying coefficients and non-reliability are taken into account. In their study, under the title of “can oil price help forecasting the U.S. stock; evidence from dynamic averaging model” Naser and Alaali (2015) studied the power of oil price and other macro-economic and macro-financial variables, including industrial production index, interest rate, inflation rate, unemployment rate and financial ratios, in predicting S&P 500.

According to empirical evidence, DMS and DMA techniques significantly improve prediction performance compared with other prediction techniques. In addition, when oil price serves as a predictor, DMS and DMA performances increase. In the most relevant study, Golarzi and Chehrrengar (2015) compared state-space performance and OLS performance in the fitness of Fama and French three factor model in order to predict returns. This study revealed that state-space model shows a better performance in forecasting returns. This may imply that the beta coefficients of Fama and French model are not constant in Tehran Stock Exchange.

This study aims to determine the time-varying volatility of internal and external variables affecting stock price return. Therefore, the factors have been analyzed by financial and economic authorities in different studies. For example, the views can be evaluated in terms of the portfolio theory of Fisher hypothesis.

Table 1 summarizes the results of different studies conducted on the effect of macro-economic variables on stock return, as well as the efficiency of time-varying volatility models, compared with that of traditional models.

According to table 1, the majority of studies have found that the volatility of macro-economic variables affects stock return. Therefore, in the process of forming optimal portfolio, investors need to pay considerable attention to such indices and their influences. In addition, this table shows that the national and international studies have discovered that the efficiency of time-varying models is higher than that of traditional ones.
Table 1. Summary of the results

<table>
<thead>
<tr>
<th>Effect of microeconomic variables on stock return</th>
<th>International studies</th>
<th>Local studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advocates</td>
<td>Opponents</td>
<td>Advocates</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Efficiency of time-varying volatility models in comparison to traditional models</th>
<th>International studies</th>
<th>Local studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advocates</td>
<td>Opponents</td>
<td>Advocates</td>
</tr>
</tbody>
</table>

3. Methodology

Time-series regression model is a conventional statistical model where the changes of a phenomenon are studied over time. Such techniques assume that an equation with constant coefficients can be used in different times. Inaccurate results originated from such a non-realistic assumption leaded to dynamic models which are very closer to the real world. State-space model is a method for modeling dynamic systems which models, predicts and analyzes the behavior of system in such conditions.

State-space models let parameters have structural instability and let coefficients be constant over time. This is one of the applications of such models. Such models are known as Time-Varying Parameter (TVP) models which is a special state of state-space models (Heidari and Salehinezhad, 2012). State-space equations system consists of two equations: observation equation and equation of state. The equations are estimated using reversible algorithms (Kalman filter or particle filter). Bayes filter is the most typical estimation method. From Bayesian theory point of view, the problem of estimation is estimating probability density function posterior. Given probability density function posterior, the optimal estimation of states can be calculated in terms of any criterion function.

There are different techniques for practical solutions of Bayes filter, depending on relevant process and measurement. For example, if the studied dynamic system is a linear system and process and measurement noises are of Gaussian nature, Kalman filter will be used. If the system is non-linear with a white Gaussian noise, extended Kalman filter will be used and if the non-linearity of system is very high, extended Kalman filter will not optimally function. Non-parametric techniques are other practical solutions for implementing Bayes filter. Particle filter is the most important non-parametric technique for estimating non-linear systems and many researchers have worked on this technique. Particle filter is a group of filters where Monte Carlo technique is used to estimate posterior distribution. This study adopts the geometric mean of the outputs of Kalman filter as well as particle filter in order to both cover the weaknesses of each filter and have an improved and more accurate prediction of stock price return using this combined filter. This section introduces methods adopted in this study.

3.1. TVP Regression with Stochastic Volatility

TVP model with stochastic volatility enables us to record the probable changes of the fundamental structure of economy more flexibly and more powerfully. According to many studies, combining stochastic volatilities with TVP estimation improves estimation performance significantly. Let us consider TVP regression model as follows:

**Regression:**

\[ y_t = x_t' \beta + x_t' \alpha_t + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_t^2), \quad t = 1, ..., n \] (1)

**Time-varying coefficients:**

\[ \alpha_{t+1} = \alpha_t + u_t \quad u_t \sim N(0, \Sigma), \quad t = 1, ..., n - 1 \] (2)
**Stochastic volatility:**
\[ \sigma_t^2 = \gamma \exp(h_t), \quad u_{t+1} = \varphi h_t + \eta_t, \quad \eta_t \sim N(0, \sigma_t^2), \quad t = 1, \ldots, n - 1 \]  

Where \( y_t \) is a scalar of response; \( x_t \) and \( z_t \) are \((k \times 1)\) and \((p \times 1)\) vectors of covariates, respectively; \( \beta \) is \((k \times 1)\) vector of constant coefficients; \( \alpha_t \) is a \((p \times 1)\) vector of Time-varying coefficients; and \( h_t \) is stochastic volatility. Stochastic volatility plays a significant role in TVP models. Although the idea of stochastic volatility was first presented by Black (1976), financial econometrics has experienced many changes (Gisel, Harvey and Reynald, 2002, Shepard, 2005).

### 3.2. Dynamic Models

The standard form of state-space models, especially that of Kalman filter, is as follows:

\[ y_t = z_t \theta_t + \epsilon_t \]  
\[ \theta_t = \theta_{t-1} + \mu_t \]  

Where \( y_t \) is the dependent variable of model, \( z_t = [1, x_{t-1}, y_{t-1}, \ldots, y_{t-p}] \) is a \( 1 \times m \) vector constituted of intercepts estimators and dependent variable interval and \( \theta_t = [\varphi_{t-1}, \beta_{t-1}, y_{t-1}, \ldots, y_{t-p}] \) is a \( 1 \times n \) vector constituted of coefficients (states). \( \epsilon_t \sim N(0, H_t) \) and \( \mu_t \sim (0, Q_t) \), which have normal distribution with zero mean, are \( H_t \) and \( Q_t \) variances, respectively. These models have many advantages the most important of which is the possibility of varying estimated coefficients at any time. The main disadvantage of such models is that if \( z_t \) gains a high value, the estimations will not be reliable. The extended TVP model has the same problems of TVP-VAR models. This model was properly developed by Garvin et al. (2008) in which the behavior uncertainties of estimators were introduced to the model as follows:

\[ y_t = \sum_{j=1}^{m} s_{j} \theta_{jt} z_{jt} + \epsilon_t \]  

Where \( \theta_{jt} \) and \( z_{jt} \) are the \( j^{th} \) element of \( \theta_t \) and \( z_t \), respectively. Their model has an additional element: the existence of \( s_j \in (0,1) \) variable. This variable cannot vary with time and serves as a permanent variable which can accept \( 1 \) and \( 0 \) for any estimator (Hoogerheide et al., 2009) Raftery et al. (2010) introduced DMA method and eliminated all restrictions of previous methods. This method could estimate large models at any instant and made it possible to change the input variables of model at any time.

In order to explain DMA process, let us assume that there are \( k \) sub-set models of \( z_t \) variables of estimators where \( z^{(k)} (k = 1, 2, \ldots, K) \) indicates \( k \) sub-set models. Based on this assumption, given \( k \) sub-set models at any time, state-space model is described as follows:

\[ y_t = z_t^{(k)} \theta_t^{(k)} + \epsilon_t^{(k)} \]  
\[ \theta_{t+1} = \theta_t^{(k)} + \mu_t^{(k)} \]  

Where \( \epsilon_t^{(k)} \sim N(0, H_t^{(k)}) \) and \( \mu_t^{(k)} \sim (0, Q_t^{(k)}) \). \( \theta_t = (\theta^{(1)}_t, \ldots, \theta^{(K)}_t) \), \( L_t \in \{1, 2, \ldots, K\} \) stands for the model, out of the \( K \) sub-set models, which best fits with a given time. That method which makes it possible to estimate a different model at a given instant is called dynamic averaging model (Koop and Kroublis, 2011). Regarding the differences between DMA and DMS dynamic models in forecasting a variable at time \( t \) based on data of time \( t-1 \), it can be argued that given \( L_t \in \{1, 2, \ldots, K\} \), DMA calculates \( Pr (L_t = k | y^{t-1}) \) and determines the average of the models predictions based on the above probability; while DMS selects a model with the highest possible probability of \( Pr (L_t = k | y^{t-1}) \) and forecasts the model with the maximum probability.

### 3.3. Evaluation of the Accuracy of Estimation Models

In order to evaluate a prediction model or to select the best fit model out of different available models for given time series, we need an index by which we can make decision about the acceptance or rejection of prediction model. This study adopts mean squared forecast error (MEFE) and mean absolute forecast error (MAFE) indices as follows:

\[ MSFE = \frac{\sum_{\tau=\tau_0}^{\tau+n}(y^\tau - E(y^\tau|Data_{\tau-n}))^2}{\tau_{t+1} - \tau_0 + 1} \]  
\[ MAFE = \frac{\sum_{\tau=\tau_0}^{\tau+n}|y^\tau - E(y^\tau|Data_{\tau-n})|}{\tau_{t+1} - \tau_0 + 1} \]  

Where \( Data_{\tau-n} \) is data derived from period \( \tau - h \) and \( h \) is forecasting time horizon and \( E(y^\tau|Data_{\tau-n}) \) is the point forecast of \( y^\tau \).
4. Results
4.1. Data Presentation
This study employed 2003-2013 data (with monthly intervals) for the variables of Tehran Stock Exchange return, non-official exchange rate change as the variable of internal market shock, interest rate (monetary policy), oil price change as the variable of foreign shock and inflation (general policy). The above variables were developed by Iranian Central Bank and International Monetary Fund, respectively. The logarithm of the ratio of Tehran stock exchange index at a given period to the previous period was multiplied by 100 and was considered as the return of Tehran Stock Exchange (Aloui and Jammazi, 2009).

\[ Y_t = 100 \times \ln \left( \frac{TEPIX_t}{TEPIX_{t-1}} \right) \]

Table 2 shows the variables used in computer-based calculations to forecast and estimate the cash return of stock exchange.

Table 2. Variables of model

<table>
<thead>
<tr>
<th>Variable names</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant sentence</td>
</tr>
<tr>
<td>1st interval of cash return</td>
</tr>
<tr>
<td>Inflation</td>
</tr>
<tr>
<td>Non-official exchange rate change</td>
</tr>
<tr>
<td>interest rate</td>
</tr>
<tr>
<td>Oil price change</td>
</tr>
</tbody>
</table>

Since it is not necessary to evaluate data durability in TVP methods, the efficiency of TVP models, compared with traditional OLS models, will be assessed using maximum likelihood value statistics. Table 3 shows the results.

Table 3. LR test for comparing the efficiencies of TVP and OLS models

<table>
<thead>
<tr>
<th></th>
<th>InL</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>260.16</td>
<td></td>
</tr>
<tr>
<td>TVP</td>
<td>272.171</td>
<td>( x^2 = 24.02^{***} )</td>
</tr>
</tbody>
</table>

*** is significant at %1 sig. level

The results of LR test, shown in table 3, indicate that TVP model has higher likelihood rate than OLS model 1% \((272.171 > 260.16)\). Therefore, TVP approaches (non-linear approaches) estimate models more efficiently than traditional OLS models (linear models).

4.2. Model Estimation
Figure 1 illustrates actual and predicted values as well as conditional variances derived from TVP estimation with stochastic volatility.

In Figure 1, the upper curve illustrates the actual and predicted values of stock return and the lower one illustrates the conditional variances of stock return out of total return series.

Figures 2 to 11 illustrate time-varying coefficients estimated by TVP model with stochastic volatility for each independent variable. In linear regression models, such as OLS model, only one coefficient (point coefficient) was calculated for each variable. In non-linear models, such as regime change models, two or three coefficients were calculated for each variable, depending on the number of regimes which is generally 2 or 3. The following curves were derived using TVP models with stochastic volatility. In this method, one coefficient is calculated for each time period. Therefore, for each coefficient of model it is possible to calculate some coefficients the number of which equals to the number of time periods. The following curves illustrate the trend of estimated coefficients, not the trend of the data of each variable, for every variable.

Table 4 shows MAPE and MSFE values estimated by different DMA, DMS and TVP models in the forecast horizon of 1 and 4.

The results shown in table 4 indicate that all models of this study have a higher accuracy than OLS model (traditional approach) where with \( \alpha = \beta = 0.90 \). DMS model has the higher forecasting accuracy than other models. Table 4 shows the estimation of the best model with \( \alpha = \beta = 0.90 \). The above model with time-varying variables can best predict the price return of Tehran Stock Exchange. According to Table 5, it is possible to identify variables affecting the return of stock exchange in any given time period. For example, for time period during the fourth month of year 2003, the first interval of stock exchange return and interest rate have affected stock return or for time period during the eleventh month of the year 2003, the first interval of stock return, inflation, exchange rate and the first interval of interest rate had the highest effect on the return of Tehran Stock Exchange, respectively. Similar analyses can be presented for other periods.
Figure 1. Actual and predicted values and conditional variances

Figure 2. Time varying parameter AR(1)

Figure 3. Time varying parameter Constant

Figure 4. Time varying parameter Inflation

Figure 5. Time varying parameter Inflation (-1)
Investigating the Impact of Time-varying Volatility of Macroeconomic Indices on the …

Table 4. Comparison of different models

<table>
<thead>
<tr>
<th>Prediction method</th>
<th>MAFE</th>
<th>MSFE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h = 1</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>11.86</td>
<td>143.21</td>
</tr>
<tr>
<td>DMA $a = \beta = 0.99$</td>
<td>7.39</td>
<td>98.27</td>
</tr>
<tr>
<td>DMA $a = \beta = 0.99$</td>
<td>6.53</td>
<td>70.86</td>
</tr>
<tr>
<td>DMA $a = \beta = 0.90$</td>
<td>6.32</td>
<td>72.2</td>
</tr>
<tr>
<td>DMA $a = \beta = 0.90$</td>
<td>5.05</td>
<td>49.68</td>
</tr>
</tbody>
</table>

Vol.1 / No.3 / Autumn 2016
The results obtained from the above table are presented below.  
- The first interval of stock return had a significant effect on stock return in all time periods (126 periods)  
- Interest rate had a significant effect on stock return in 36 periods  
- The first interval of interest rate had a significant effect on stock return in 32 periods  
- Inflation rate had a significant effect on stock return in 23 periods
Investigating the Impact of Time-varying Volatility of Macroeconomic Indices on the...

- The first interval of inflation rate had a significant effect on stock return in 16 periods
- Oil price had a significant effect on stock return in 42 periods
- The first interval of oil price had a significant effect on stock return in 52 periods
- Exchange rate had a significant effect on stock return in 38 periods
- The first interval of exchange rate had a significant effect on stock return in 16 periods

The final conclusion reveals that oil price and exchange rates had the highest effects on stock returns in the study period proceeded only by the first interval of stock return. According to estimations of model, shown in table 5, the results of this study confirms the theoretical dimensions of CAPM models, arbitrage and portfolio in Tehran Stock Exchange. Therefore, it can be argued that in addition to non-systematic risks, systematic risks affect the prediction of the stock return of Iranian Stock Exchange.

5. Discussion and Conclusions

The results of this study indicated the higher accuracy of dynamic models with time-varying parameters in predicting the return of stock exchange, i.e. values of MAFE and MSFE in different DMA and DMS models, which are completely dynamic models, are higher than that of TVP models. The results of this study revealed that the mere variability of the coefficients of variables in TVP models cannot improve the simulation accuracy of price return of stock exchange and the assumption of the dynamism of input variables of model is an important factor which improves the simulation accuracy of stock exchange return. According to results, at different time periods, variables with different intensities (different coefficients) affect stock return. On this basis, the effect of oil price and exchange rate variables are higher than that of interest rate and inflation rate. It can be argued, therefore, that the impact of time-varying volatility of internal variables (interest rate and inflation rate) is lower on stock return than that of time-varying volatility of external variables (oil price and exchange rate). The results of this study are in congruence with similar results including the results of the studies conducted by Naser and Alaali (2015), Chan et al. (2015), Johannes et al. (2014), Fux (2014), Nakajima (2011) and Wang et al. (2016), Shojaei (2013), and Zolfaghari (2011).

According to the study findings, since different variables have different impacts on stock return at different times, it is recommended to predict stock return using models with the ability of separating regime changes at different likelihood levels. To this end, the policy developers and the authorities of financial markets are recommended not to implement general policies at all times in order to improve the status of financial markets. They are recommended to formulate policies for each regime using the appropriate instruments depending on the most important influential factors of stock return of that regime. It should be noted that the prediction of stock return using state-space model is a widespread field. Therefore, to complete the chain of studies on the models the interested researchers are recommended to take other influential factors into account and estimate stock return in stock exchange market by extending study scope to other variables such as sanctions or by implementing specific policies such as targeted subsidies or executing structural reforms.

References

5) Chan, J. C. C., Eisenstat, E. (2015). Bayesian model comparison for time-varying parameter VARs with stochastic volatility, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, Australian National University.


Investigating the Impact of Time-varying Volatility of Macroeconomic Indices on the 


